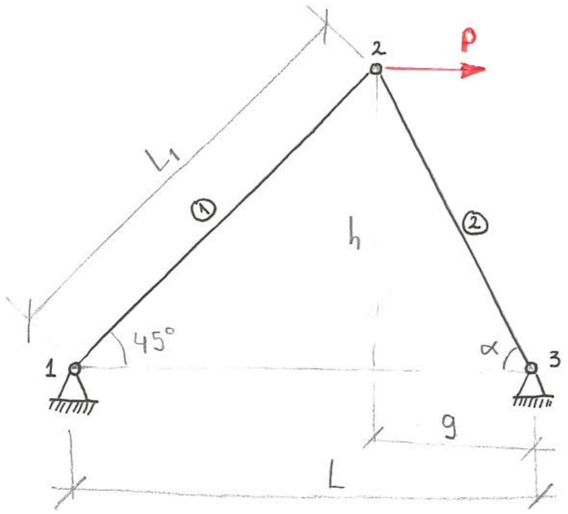
FINITE ELEMENT METHOD



Where: $E = 207\ GPa$

$A_1 = 10\ cm^2$

$A_2 = 20\ cm^2$

$L = 1.268\ m$

$L_1 = \sqrt{2}\ m$

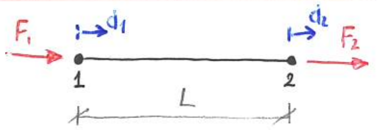
$P = 100\ KN$

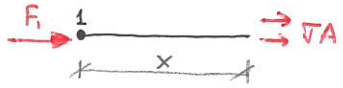
Preliminary calculations

$\cos 45° = \dfrac{L-g}{L_1} \longrightarrow g = 0.268\,m$; $\sin 45° = \dfrac{h}{L_1} \longrightarrow h = 1\,m$

$\tan\alpha = \dfrac{h}{g} \longrightarrow \alpha = 74.997° \simeq 75°$; $L_2 = \sqrt{g^2+h^2} = 1.035\,m$

Bar in local coordinate system





$$\left.\begin{array}{l} F_1 + \sigma A = 0 \\ \left.\begin{array}{l} \sigma = E\varepsilon \\ \varepsilon = \dfrac{du}{dx} \end{array}\right] \longrightarrow \sigma = E\dfrac{du}{dx} \end{array}\right] \Rightarrow F_1 + EA\dfrac{du}{dx} = 0 \Rightarrow F_1 x + EAu + C_1 = 0 \quad (I)$$

$x = 0 \longrightarrow u = d_1$: $EAd_1 + C_1 = 0 \longrightarrow C_1 = -EAd_1$

Substituting in (I): $F_1 x + EAu - EAd_1 = 0$ (II)

$x = L \longrightarrow u = d_2$: $F_1 L + EAd_2 - EAd_1 = 0 \longrightarrow \boxed{F_1 = \dfrac{EA}{L}(d_1 - d_2)}$

Applying equilibrium: $\boxed{F_2 = -F_1 = \dfrac{EA}{L}(-d_1 + d_2)}$

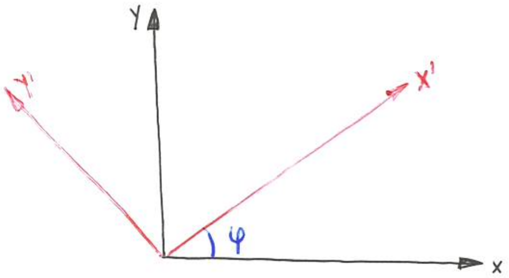
Therefore:

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \Rightarrow \boxed{\{F\} = [K]\{d\}}$$

Bar 1 (local coordinate system)

$$K_1' = \frac{EA_1}{L_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{EA_1}{L_1}\begin{array}{c} \begin{array}{cccc} d_{1x'} & d_{1y'} & d_{2x'} & d_{2y'} \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Bar 1 (global coordinate system)



$$\left.\begin{array}{l} i = \cos\varphi\, i' - \sin\varphi\, j' \\ j = \sin\varphi\, i' + \cos\varphi\, j' \end{array}\right] \qquad T = \begin{bmatrix} \cos\varphi & +\sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

$$T^{-1} = T^T$$

$$K_1' = T K_1 T^T \longrightarrow K_1 = T^T K_1' T \quad (\text{III})$$

$$T = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 & 0 \\ -\sin\varphi & \cos\varphi & 0 & 0 \\ 0 & 0 & \cos\varphi & \sin\varphi \\ 0 & 0 & -\sin\varphi & \cos\varphi \end{bmatrix}$$

Following equation (III):

$$K_1 = \frac{EA_1}{L_1}\begin{bmatrix} \cos^2\varphi & \sin\varphi\cos\varphi & -\cos^2\varphi & -\sin\varphi\cos\varphi \\ \sin\varphi\cos\varphi & \sin^2\varphi & -\sin\varphi\cos\varphi & -\sin^2\varphi \\ -\cos^2\varphi & -\sin\varphi\cos\varphi & \cos^2\varphi & \sin\varphi\cos\varphi \\ -\sin\varphi\cos\varphi & -\sin^2\varphi & \sin\varphi\cos\varphi & \sin^2\varphi \end{bmatrix}$$

When $\varphi = 45°$:

$$K_1 = 73{,}185{,}551.85 \begin{array}{c} \begin{array}{cccc} d_{1x} & d_{1y} & d_{2x} & d_{2y} \end{array} \\ \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

Bar 2 (local coordinate system)

$$K_2' = \frac{EA_2}{L_2}\begin{array}{c} \begin{array}{cccc} d_{2x} & d_{2y} & d_{3x} & d_{3y} \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Bar 2 (global coordinate system)

Following the process intruduced for bar 1:

$$K_2 \doteq 4\cdot 10^8 \begin{array}{c} \begin{array}{cccc} d_{2x} & d_{2y} & d_{3x} & d_{3y} \end{array} \\ \begin{bmatrix} 0.067 & 0.25 & -0.067 & -0.25 \\ 0.25 & 0.933 & -0.25 & -0.933 \\ -0.067 & -0.25 & 0.067 & 0.25 \\ -0.25 & -0.933 & 0.25 & 0.933 \end{bmatrix} \end{array}$$

Assembly

$$
K_T = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}
\begin{array}{cccccc} d_{1x} & d_{1y} & d_{2x} & d_{2y} & d_{3x} & d_{3y} \end{array}
$$

$$
K_T = \begin{bmatrix}
A & A & -A & -A & 0 & 0 \\
A & A & -A & -A & 0 & 0 \\
-A & -A & B & C & -F & -G \\
-A & -A & C & D & -G & -H \\
0 & 0 & -F & -G & F & G \\
0 & 0 & -G & -H & G & H
\end{bmatrix}
$$

where: $A = 73,185,551.85$

$B = 99,985,551.85$

$C = 173,185,551.9$

$D = 446,385,551.9$

$F = 26,800,000$

$G = 100,000,000$

$H = 373,200,000$

System

$$
\begin{bmatrix}
A & A & -A & -A & 0 & 0 \\
A & A & -A & -A & 0 & 0 \\
-A & -A & B & C & -F & -G \\
-A & -A & C & D & -G & -H \\
0 & 0 & -F & -G & F & G \\
0 & 0 & -G & -H & G & H
\end{bmatrix}
\begin{bmatrix} 0 \\ 0 \\ d_{2x} \\ d_{2y} \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix} F_{1x} \\ F_{1y} \\ P \\ 0 \\ F_{3x} \\ F_{3y} \end{bmatrix}
$$

Note that: $d_{1x} = d_{1y} = d_{3x} = d_{3y} = 0$ because of the boundary conditions.

$-A\,d_{2x} - A\,d_{2y} = F_{1x}$ (IV)

$-A\,d_{2x} - A\,d_{2y} = F_{1y}$ (V)

$B\,d_{2x} + C\,d_{2y} = P$ (VI)

$C\,d_{2x} + D\,d_{2y} = 0$ (VII)

$-F\,d_{2x} - G\,d_{2y} = F_{3x}$ (VIII)

$-G\,d_{2x} - H\,d_{2y} = F_{3y}$ (IX)

From (VII): $d_{2x} = -\frac{D}{C} d_{2y}$ (X)

Substituting (X) in (VI): $\left(-\frac{BD}{C} + C\right) d_{2y} = P \Rightarrow \boxed{d_{2y} = -1.183 \cdot 10^{-3}\, m}$

Using the value of d_{2y} in (X): $\boxed{d_{2x} = 3.05 \cdot 10^{-3}\, m}$

Therefore:

$$
\boxed{\begin{aligned}
F_{1x} &= -136,637.425 \text{ N} \\
F_{1y} &= -136,637.425 \text{ N} \\
F_{3x} &= 36,650 \text{ N} \\
F_{3y} &= 136,495.6 \text{ N}
\end{aligned}}
$$